

ETIC FILE COPY

REPORT SD-TR-89-20

(4)

## A Method for Calculating Asymmetric Flow Through Nozzles

Prepared by

RUDOLF X. MEYER  
Engineering Group  
The Aerospace Corporation  
El Segundo, CA 90245-4691

January 1989

Prepared for

SPACE DIVISION  
AIR FORCE SYSTEMS COMMAND  
Los Angeles Air Force Base  
P.O. Box 92960  
Los Angeles, CA 90009-2960

DTIC  
ELECTE  
JUN 26 1989  
S E D



THE AEROSPACE CORPORATION

APPROVED FOR PUBLIC RELEASE;  
DISTRIBUTION UNLIMITED

29 6 26 038

AD-A209 689

This final report was submitted by The Aerospace Corporation, El Segundo, CA 90245-4691, under Contract No. F04701-88-C-0089 with the Space Division, P. O. Box 92960, Los Angeles, CA 90009-2960. It was reviewed and approved for The Aerospace Corporation by Dave Griep, Vice President, Engineering. The project officer is Lt. Col. Holljes SD/CLMD.

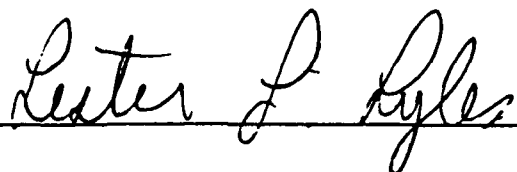
This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.



---

FOR THE COMMANDER



---

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

## REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION <b>Unclassified</b>			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION / AVAILABILITY OF REPORT  Approved for public release; distribution unlimited		
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S)  TR-0088(4392-11)-1			5. MONITORING ORGANIZATION REPORT NUMBER(S)  SD-TR-89-20		
6a. NAME OF PERFORMING ORGANIZATION  The Aerospace Corporation		6b. OFFICE SYMBOL (if applicable)	7a. NAME OF MONITORING ORGANIZATION Space Division Air Force Systems Command		
6c. ADDRESS (City, State, and ZIP Code)  2350 E. El Segundo Blvd. El Segundo, CA 90245-4691			7b. ADDRESS (City, State, and ZIP Code) Los Angeles Air Force Base P. O. Box 92960 Los Angeles, CA 90009-2960		
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Space Division		8b. OFFICE SYMBOL (if applicable) CLMD	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER  F04701-88-C-0089		
8c. ADDRESS (City, State, and ZIP Code)  See 7b.			10. SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.
					WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification)  A Method for Calculating Asymmetric Flow Through Nozzles					
12. PERSONAL AUTHOR(S) R. X. Meyer					
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM 1-88 TO 8-88		14. DATE OF REPORT (Year, Month, Day) January 1989	
15. PAGE COUNT 29					
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP			
			Rocket nozzles; asymmetric flow; integral method; oblique flow.		
19. ABSTRACT (Continue on reverse if necessary and identify by block number)  The asymmetric steady-state, inviscous, adiabatic flow of a perfect gas through a subsonic/supersonic nozzle is considered. The nozzle itself is either of rectangular cross section or is axisymmetric. A first-order small asymmetry induced by an entrance flow which is oblique to the nozzle axis and has a transverse pressure gradient is allowed. The cross section of the nozzle is assumed to vary only slowly as a function of the axial distance; the case considered here is therefore an extension of the well-known theory of the quasi-one-dimensional flow of a perfect gas treated in standard textbooks. An integral method is used to obtain approximate results. The method is simple, yet in a test case (supersonic flow at an angle of attack through a rectangular channel) where the exact first-order result is known, a comparison shows surprisingly good agreement. Numerical results are tabulated for axisymmetric nozzles with polynomial Mach number dependence.					
20. DISTRIBUTION / AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION <b>Unclassified</b>		
22a. NAME OF RESPONSIBLE INDIVIDUAL			22b. TELEPHONE (Include Area Code)		22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted.  
All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

# ACKNOWLEDGMENT

The author wishes to thank Wing Yeung for having carried out the calculations and for valuable discussions.

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	



## CONTENTS

1.	INTRODUCTION .....	7
2.	CONSERVATION EQUATIONS .....	11
3.	INTEGRAL METHOD .....	15
4.	A TEST CASE .....	19
5.	NOZZLES WITH POLYNOMIAL MACH NUMBER DEPENDENCE .....	21
	REFERENCES .....	27
	NOMENCLATURE .....	29

## FIGURES

1.	Supersonic Flow with Small Angle of Incidence Through Rectangular Duct .....	19
2.	Nozzle Contours Satisfying Eq. (10), for $\gamma = 1.20$ .....	21
3.	Function $k_1(\zeta)$ Computed from Eq. (9) and Normalized to $k_1 = 1$ at Nozzle Throat .....	23
4.	Nondimensional Ratio $F_1(\zeta)/(p^*R^*)$ of the Transverse Force $F_1$ on the Nozzle Wall, per Unit Axial Length .....	23

## TABLES

1.	Function $F_1(z)/(p^*R^*)$ for the Nozzle Contour Shown in Figure 2 for $\gamma = 1.20$ .....	24
2.	Ratio $F_{1e}/F_1^*$ of the Transverse Force $F_{1e}$ on the Nozzle Wall, per Unit Axial Length, at the Nozzle Exit Plane, Relative to the Corresponding Force $F_1^*$ at the Throat .....	26

## 1. INTRODUCTION

High-speed flows that have a small lateral asymmetry occur in rocket motors that have canted nozzles (to avoid impingement of the plume on adjacent structures) or nozzles that can be vectored. Interest in this type of flow has been renewed recently, since it is thought to play a role in an instability observed in certain spin-stabilized, solid-propellant, upper-stage rockets. In this case, the flow at the nozzle entrance is oblique to the nozzle axis as a consequence of the Coriolis force acting on the combustion gas in a precessing and nutating rocket (Ref. 1,2).

Nonaxisymmetric flows through nozzles have also been considered by Walters, (Ref. 3) both experimentally and theoretically. However, in that case, the nozzles themselves were nonaxisymmetric, having an obliquely machined throat section. Other authors have studied nonaxisymmetric flows in nozzles, but generally they have confined themselves to the supersonic part of the flow, computed by the method of characteristics, starting from an assumed inclined sonic surface at the nozzle throat (Refs. 4,5). None of these studies is directly applicable to the case encountered in canted nozzles or in unstable spinning and precessing rocket vehicles.

In these applications, the flow--as a correction to the axisymmetric flow--is sufficiently closely approximated by assuming a steady-state, inviscous and adiabatic flow of a perfect gas. Based on the assumption of constant reservoir conditions upstream of the nozzle, the flow is isentropic and isoenergetic (constant total enthalpy) everywhere. Analogously to the well-known, elementary theory of quasi-one-dimensional flow, the nozzle cross section is assumed to vary only slowly with axial distance. Since the lateral asymmetry caused by the angle of attack or transverse pressure gradient of the entrance flow is also assumed small, the effects due to the changing cross section and those due to the lateral asymmetry can both be treated as small perturbations superposed on the one-dimensional zero-order flow.

and similarly for the density  $\rho$ , enthalpy  $h$ , and the Cartesian velocity components  $u, v, w$ ; hence, for instance, for  $u$  we write

$$u = u_0(z) + \epsilon u_1 + \text{s.f.p.} \quad (1b)$$

The symmetric flow perturbation (s.f.p.) terms are written informally merely as a reminder; it will be clear from the subsequent development that, by reason of symmetry, they do not contribute to any of the integrals defined below.

The first-order asymmetric terms  $\rho_1, h_1, w_1$  can all be expressed in terms of  $p_1$ . Thus, from the assumption of a perfect gas and constant entropy,  $\rho/\rho_0(z) = [p/p_0(z)]^{1/\gamma}$  where  $\gamma$  is the ratio of the specific heats. Since this relation applies separately to the symmetric and the asymmetric case and superposition holds, it follows, after neglecting second- and higher-order terms, that

$$\rho_1/\rho_0(z) = \frac{1}{\gamma} p_1/p_0(z) \quad (2a)$$

and similarly

$$h_1/h_0(z) = \frac{\gamma - 1}{\gamma} p_1/p_0(z) \quad (2b)$$

From conservation of energy,  $h + 1/2(u^2 + v^2 + w^2) = \text{const.}$ , and neglecting again second-order terms, it follows that  $w_1/w_0 = -h_1/w_0^2$ . Introducing the Mach number  $M_0(z)$  of the quasi-one-dimensional flow,  $M_0 = w_0/(\gamma - 1)h_0$ , we can express the asymmetric perturbation of the axial velocity component by

$$w_1/w_0(z) = - [\gamma M_0^2(z)]^{-1} p_1/p_0(z) \quad (2c)$$



## 2. CONSERVATION EQUATIONS

We consider a control volume bounded by transverse planes at  $z$  and  $z + dz$  and by the nozzle walls. We designate by  $\epsilon P_{1x}(z)$  the transverse momentum (which is in the  $x$ -direction) carried by the flow through the transverse plane, per unit time. In the case of the rectangular nozzle,  $P_{1x}$  is taken per unit width in the  $y$ -direction. From conservation of momentum to the lowest significant order

$$\frac{dP_{1x}}{dz} = - \left[ (P_1)_{x=R(z)} - (P_1)_{x=-R(z)} \right] \quad \text{if } \sigma = 1 \quad (3a)$$

$$\frac{dP_{1x}}{dz} = -R(z) \int_{\phi=0}^{2\pi} (P_1)_{r=R(z)} \cos\phi \, d\phi \quad \text{if } \sigma = 2 \quad (3b)$$

for the rectangular ( $\sigma = 1$ ) and axisymmetric ( $\sigma = 2$ ) nozzle, respectively.

The transverse angular momentum about the origin of the coordinate system is in the  $y$ -direction. The amount carried through a plane normal to the nozzle axis, per unit time (and per unit width in the case of the rectangular nozzle), is designated by  $\epsilon L_{1y}(z)$ . When second- and higher-order terms are dropped, the momentum flux  $\rho w^2$  in the  $z$ -direction becomes

$$\rho w^2 = \rho_0 w_0^2 + 2\epsilon \rho_0 w_0 w_1 + \epsilon w_0^2 \rho_1 + \text{s.f.p.}$$

By symmetry, the zero and symmetric first-order terms in  $\rho w^2$  make no contribution to the integration over  $x$ , so that for the rectangular nozzle

$$L_{1y} = z P_{1x} - \int_{x=-R(z)}^{R(z)} (2\rho_0 w_0 w_1 + w_0^2 \rho_1) x dx \quad \text{if } \sigma = 1 \quad (4a)$$

The first term on the right comes from the  $u$  component of the velocity, integrated over the cross section; the second term is from  $w$ .

By essentially the same argument, for the axisymmetric nozzle

$$L_{1y} = zP_{1x} - \int_{r=0}^{R(z)} \int_{\phi=0}^{2\pi} (2\rho_0 w_0 w_1 + w_0^2 \rho_1) r^2 \cos\phi \, dr \, d\phi \quad \text{if } \sigma = 2 \quad (4b)$$

Conservation of angular momentum then requires that

$$\frac{dL_{1y}}{dz} = \frac{d}{dz} \int_{x=-R(z)}^{R(z)} p_1 x dx + \left( z + R \frac{dR}{dz} \right) \frac{dP_{1x}}{dz} \quad \text{if } \sigma = 1 \quad (5a)$$

$$\frac{dL_{1y}}{dz} = \frac{d}{dz} \int_{r=0}^{R(z)} \int_{\phi=0}^{2\pi} p_1 r^2 \cos\phi \, dr \, d\phi + \left( z + R \frac{dR}{dz} \right) \frac{dP_{1x}}{dz} \quad \text{if } \sigma = 2 \quad (5b)$$

where Eqs. (3a) and (3b), respectively, have been used and where again the zero-order and first-order symmetric terms make no contribution to the integrals. The first term on the right represents the moment from forces acting on the transverse planes bounding the control volume; the last term results from the wall pressure.

The term  $L_{1y}$  is eliminated from these equations by differentiating Eqs. (4) and substituting the result into Eqs. (5). When Eqs. (2) also are used to eliminate  $\rho_1$  and  $w_1$ , it follows that

$$\frac{d}{dz} \int_{x=-R(z)}^{R(z)} (M_0^2 - 1) p_1 x dx + R \frac{dR}{dz} \frac{dP_{1x}}{dz} - P_{1x} = 0 \quad \text{if } \sigma = 1$$

$$\frac{d}{dz} \int_{r=0}^{R(z)} \int_{\phi=0}^{2\pi} (M_0^2 - 1) p_1 r^2 \cos\phi \, dr \, d\phi + R \frac{dR}{dz} \frac{dP_{1x}}{dz} - P_{1x} = 0 \quad \text{if } \sigma = 2$$

where use has been made of the expression for the dynamic pressure of the quasi-one-dimensional flow (Ref. 6)  $(p_0/2)w_0^2 = (\gamma/2)p M_{0,0}^2$ . Differentiating again with respect to  $z$  and using Eqs. (3) gives the following equations for  $p_1$ , where now all other first-order perturbation terms have been eliminated:

$$\begin{aligned} & \frac{d^2}{dz^2} \left\{ \left( M_0^2 - 1 \right) \int_{x=-R(z)}^{R(z)} p_1 x dx \right\} - R \frac{dR}{dz} \frac{d}{dz} \left[ (p_1)_{x=R(z)} - (p_1)_{x=-R(z)} \right] \\ & + \left[ 1 - \frac{d}{dz} \left( R \frac{dR}{dz} \right) \right] \left[ (p_1)_{x=R(z)} - (p_1)_{x=-R(z)} \right] = 0 \quad \text{if } \sigma = 1 \quad (6a) \end{aligned}$$

$$\begin{aligned} & \frac{d^2}{dz^2} \left\{ \left( M_0^2 - 1 \right) \int_{r=0}^{R(z)} \int_{\phi=0}^{2\pi} p_1 r^2 \cos \phi \, dr \, d\phi \right\} - R \frac{dR}{dz} \frac{d}{dz} \left[ R \int_{\phi=0}^{2\pi} (p_1)_{r=R(z)} \cos \phi \, d\phi \right] \\ & + \left[ 1 - \frac{d}{dz} \left( R \frac{dR}{dz} \right) \right] R \int_{\phi=0}^{2\pi} (p_1)_{r=R(z)} \cos \phi \, d\phi = 0 \quad \text{if } \sigma = 2 \quad (6b) \end{aligned}$$



### 3. INTEGRAL METHOD

Integral methods are typically based on prescribing for the dependent variable a simple functional form that may depend on one or several parameters and satisfies the boundary conditions. The parameters then are determined such that the integral relations—in this case, the conservation equations for the transverse momentum and angular momentum integrated over the nozzle cross-section—are satisfied. With a judicious choice of the functional dependence, useful results, although of limited accuracy, can often be obtained.

In the present case, we prescribe for the asymmetric perturbation term  $p_1$  a linear dependence on the transverse coordinate, in the form therefore

$$\frac{p_1}{p_0(z)} = \frac{g_1(z)}{R(z)} \quad (7)$$

where the nondimensional coefficient  $g_1(z)$  is to be determined.

We designate by  $R^*$  the nozzle half width ( $\sigma = 1$ ) or nozzle radius ( $\sigma = 2$ ) at the throat (a fictitious throat if the quasi-one-dimensional flow is subsonic throughout) where  $M_0 = 1$ , and  $p_0 = p_0^*$  and define the nondimensional quantities

$$\xi = R/R^* \quad , \quad \zeta = z/R^* \quad , \quad k_1(\zeta) = \frac{R^*}{R(z)} \frac{p_0(z)}{p_0^*} g_1(z) \quad (8)$$

Carrying out the integrations in Eqs. (6) yields a combined relation for the rectangular and axisymmetric nozzles, in the form of a second-order differential equation for  $k_1(\zeta)$

$$\frac{1}{\sigma + 2} \frac{d^2}{d\zeta^2} \left[ \xi^{\sigma+2} \left( M_0^2 - 1 \right) k_1 \right] - \frac{d}{d\zeta} \left[ \frac{d\xi}{d\zeta} \xi^{\sigma+1} k_1 \right] + \xi^\sigma k_1 = 0 \quad (9)$$

Equation (9) represents the principal result of Sections 1 through 3 and is suitable for computer programming of the direct problem (prescribed zero order Mach number as a function of axial distance) or of the inverse problem (prescribed nozzle contour). In either case, it is advantageous to express the function  $\xi(\zeta)$  in Eq. (9) by the Mach number  $M_o(\zeta)$ . From a well-known formula for quasi-one-dimensional flows (Ref. 6)

$$\xi = \left\{ \frac{1}{M_o^2} \left[ \frac{2}{\gamma + 1} \left( 1 + \frac{\gamma - 1}{2} M_o^2 \right) \right]^{(\gamma+1)/(\gamma-1)} \right\}^{1/(2\sigma)} \quad (10)$$

As one would expect, Eq. (9) exhibits the transition at  $M_o = 1$  between an essentially exponential behavior of  $k_1(\zeta)$  and, on the other hand, a wavelike character as the sign of the coefficient of the highest (second) derivative changes.

#### Boundary Conditions

As is well known, classical one-dimensional nozzle theory requires the Mach number to be unity at a throat where a transition from subsonic to supersonic flow occurs. An analogous condition occurs in the present case and is caused by the vanishing of the coefficient in Eq. (9) of the second derivative of  $k_1(\zeta)$  at  $M_o = 1$ . Carrying out the differentiations indicated in Eq. (9) and letting at the throat  $M_o = 1$ ,  $\xi = 1$ ,  $d\xi/d\zeta = 0$  results in the condition for the solution to be regular at  $\zeta = 0$

$$\frac{2}{\sigma + 2} \frac{dM_o^2}{d\zeta} \frac{dk_1}{d\zeta} + \left[ 1 - \frac{d^2}{d\zeta^2} \left( \xi - \frac{M_o^2}{\sigma + 2} \right) \right] k_1 = 0 \quad \text{at } \zeta = 0 \quad (11a)$$

This condition, therefore, relates the first derivative of  $k_1$  to  $k_1$  at a sonic throat.

A second boundary condition results from prescribing the transverse pressure gradient  $\epsilon \partial p_1 / \partial x$  at an initial (upstream) location  $\zeta = \zeta_i$ , hence the condition

$$k_1 = \frac{R^*}{p_0} \frac{\partial p_1}{\partial x} \quad \text{at } \zeta = \zeta_i \quad (11b)$$

In applications to rocket motors, the transverse pressure gradient at the entrance to the nozzle needs to be determined from pressure matching with the flow field in the motor chamber adjacent to the nozzle. Since the combustion gas velocity in the chamber is typically far below the speed of sound, the calculation is simplified by the assumption of incompressibility, although it is sometimes complicated by the geometry of the boundaries. This latter calculation is outside the scope of the present report.

After  $k_1(\zeta)$  has been obtained from a solution of Eq. (9), the transverse pressure gradient is obtained from Eqs. (7) and (8). In turn, one obtains the asymmetric force  $\epsilon F_1$  (per unit axial length) exerted by the gas on the nozzle wall. This force is often the quantity of greatest interest in applications. For instance, for the axisymmetric nozzle, after integrating over the polar angle  $\phi$

$$F_1 = \pi (p_0^* / R^*) R^2(\zeta) k_1(\zeta) \quad (12)$$

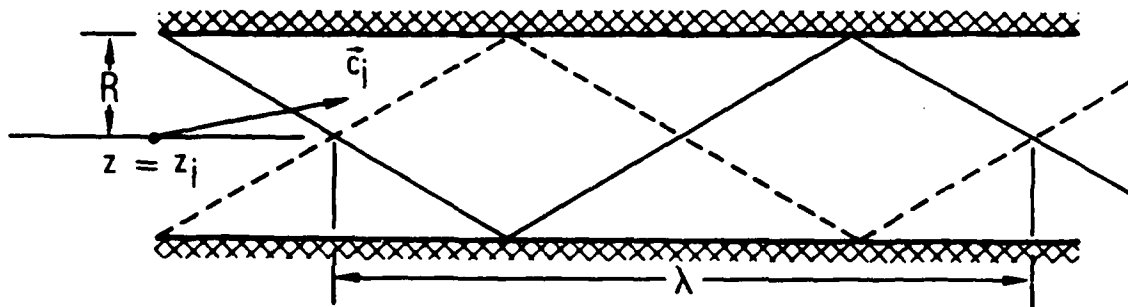
Numerical results for the homogeneous case, applied to a parameterized family of nozzle contours, are discussed in Section 5.





#### 4. A TEST CASE

Supersonic flow, incident at a small angle on a rectangular duct of uniform width (Fig. 1), represents an example in which an exact--at least in the sense of a first-order perturbation result--solution is easily calculated. It is of interest, therefore, to compare, as a test case, results obtained from Eq. (9) with the exact, but more restricted, solution in this special case. The exact first-order solution, which is well known, is described by a periodic pattern of triangular and rhombic regions bounded by Mach lines, in which the flow properties are constant, but changing discontinuously across the Mach lines.



Note:  $R$  = half width;  $C_i$  = incident flow velocity;  $\lambda$  = period

Figure 1. Supersonic Flow with Small Angle of Incidence Through Rectangular Duct

With  $M_0$  again designating the unperturbed Mach number, and  $R$  the half width, the period  $\lambda$  of the flow perturbation is given by

$$\lambda = 4R\sqrt{M_0^2 - 1} \quad (\text{exact first-order theory}) \quad (13a)$$

On the other hand, from Eq. (9), since here  $d\xi/d\zeta = dM_0^2/d\zeta = 0$  and  $\sigma = 1$

$$\frac{d^2 k_1}{d\zeta^2} + \frac{3}{\xi^2(M_0^2 - 1)} k_1 = 0$$

The transverse pressure gradient, and similarly the other perturbation quantities, therefore, have a sinusoidal dependence on the axial coordinate, with period

$$\lambda = \frac{2\pi}{\sqrt{3}} R\sqrt{M_0^2 - 1} = 3.628 R\sqrt{M_0^2 - 1} \quad (\text{integral method}) \quad (13b)$$

The integral method, therefore, gives the correct Mach number dependence although with a multiplier which differs from the correct one by approximately 10%.

## 5. NOZZLES WITH POLYNOMIAL MACH NUMBER DEPENDENCE

As an example of the application of Eq. (9), we consider an axisymmetric Laval nozzle for which the square of the unperturbed (quasi-one-dimensional) Mach number varies with axial distance as a second-degree polynomial, for which we choose

$$M_0^2 - 1 = a\zeta(1 - b\zeta), \quad a > 0, b \geq 0 \quad (14)$$

where  $a$  and  $b$  are constants. The nozzle contour is easily calculated from Eq. (10). For suitably chosen constants, relation (14) results in quite realistic nozzle contours (Fig. 2).

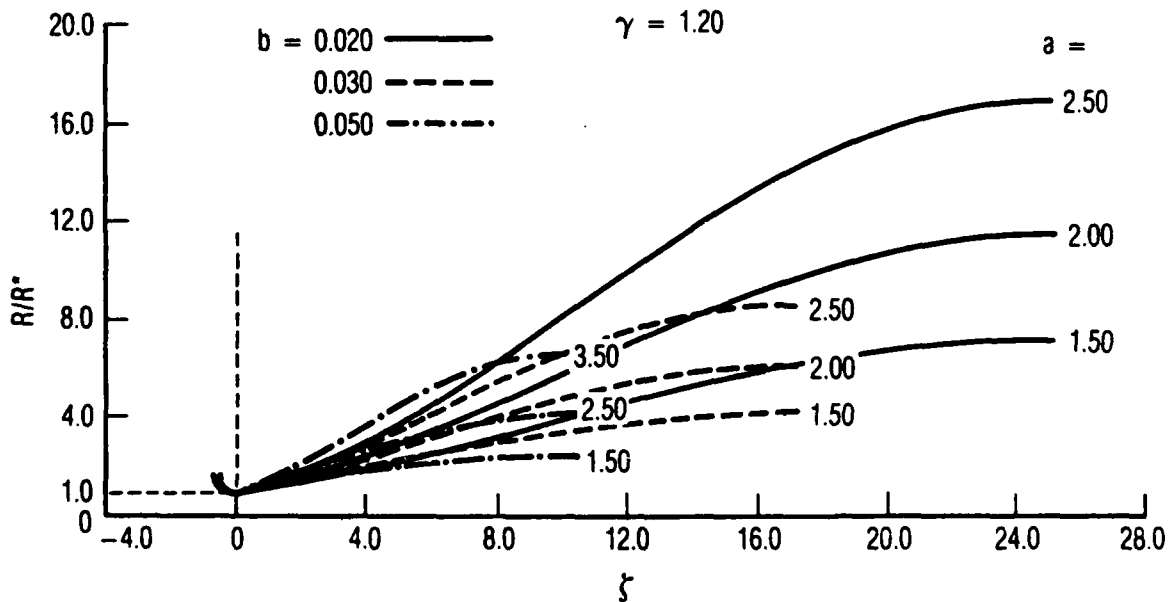


Figure 2. Nozzle Contours Satisfying Eq. (14), for  $\gamma = 1.20$

At  $M_0 = 0$  the slope of the nozzle contour becomes infinite. This occurs at  $\zeta = -(2b)^{-1}(\sqrt{1 + 4b/a} - 1)$ . Clearly, the assumption of quasi-one-dimensional flow breaks down for  $\zeta$  approaching this value. At  $\zeta = (2b)^{-1}$ , the Mach number  $M_0$  and, hence, the nozzle radius each reach a maximum, which occurs in the supersonic part of the flow.

The transverse pressure gradient in the form of the function  $k_1(\zeta)$  and the lateral force per unit axial length exerted on the nozzle, in the form of the nondimensional ratio  $F_1(\zeta)/(p^*R^*)$ , are computed for several values of the nozzle parameters  $a$  and  $b$ . They are graphed in Figures 3 and 4. (For the range of  $b$  that is of practical interest for realistic nozzle contours, the curves in Figure 3 very nearly coincide and depend only on the parameter  $a$ .) The solutions are computed for a ratio of the specific heats  $\gamma = 1.20$ , a value that is representative of many rocket motor combustion gases, and are normalized so that  $k_1 = 1$  at the nozzle throat. It is evident that the largest transverse pressure gradients and nozzle side forces occur in the subsonic and transonic sections of the nozzle where most of the readjustment of the flow direction takes place. It is also evident, particularly for the smaller values of the parameter  $a$ , that the transverse pressure gradient downstream of the throat at first reverses sign, an indication of the reflection on the nozzle walls of the Mach cones associated with the turning of the flow.

Table 1 lists numerical values for the side force on the nozzle. The effect of Mach cone reflections reversing the sign of the force in the diverging (supersonic) part of the nozzle is again evident.

In some applications, such as in spin-stabilized solid-propellant rockets, the condition at the nozzle exit plane is of particular interest, since it determines the angular momentum flux of the combustion gas and hence the resulting moment about the vehicle's center of mass. Table 2, therefore, lists the transverse force  $F_{1e}$  at the exit plane, per unit axial length, relative to the force  $F_1^*$  at the nozzle throat, for nozzles with the polynomial Mach number dependence (14) and zero nozzle divergence at the exit plane.

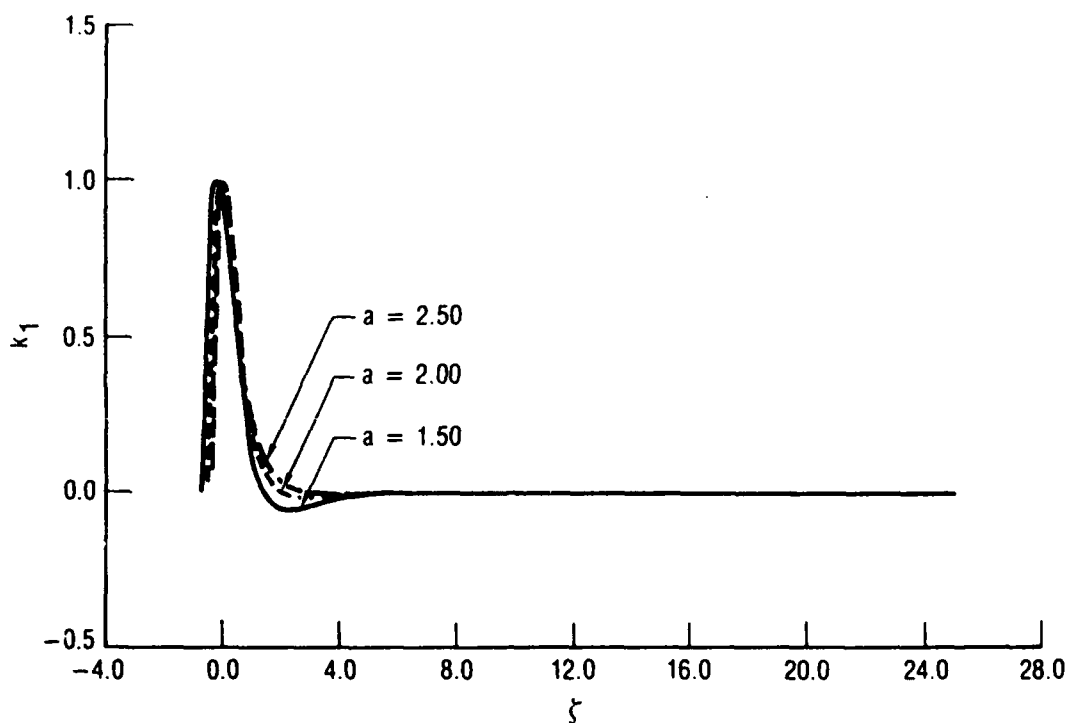


Figure 3. Function  $k_1(\zeta)$  Computed from Eq. (9) and Normalized to  $k_1 = 1$  at Nozzle Throat

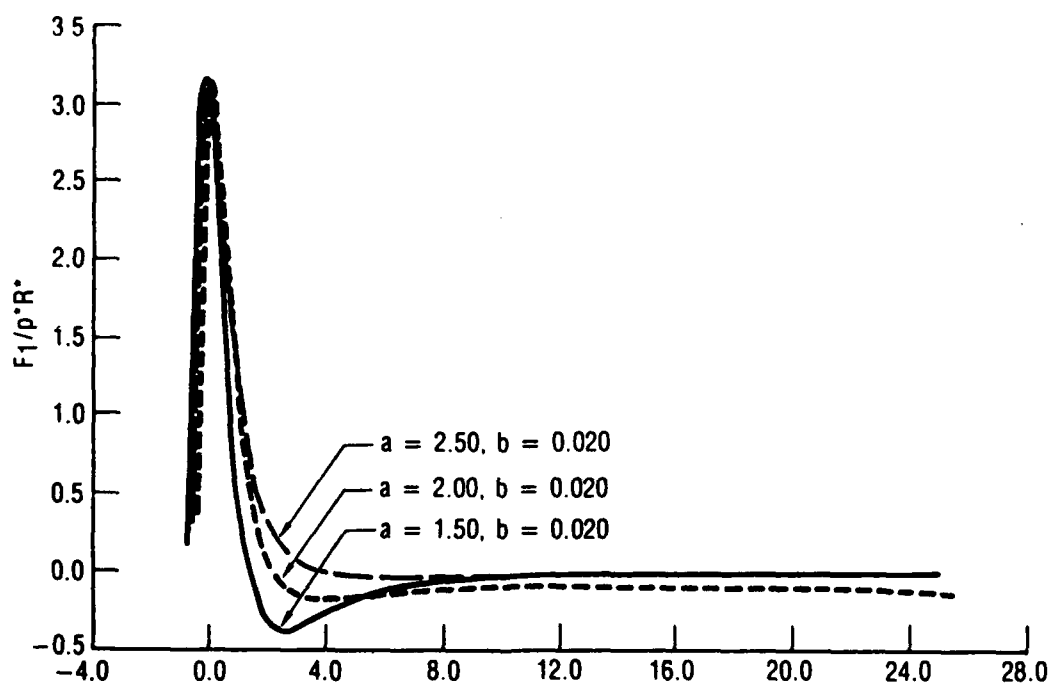


Figure 4. Nondimensional Ratio  $F_1(\zeta)/(p^*R^*)$  of the Transverse Force  $F_1$  on the Nozzle Wall, per Unit Axial Length

Table 1. Function  $F_1(\zeta)/(p^*R^*)$  for the Nozzle Contours Shown in Figure 2 for  $\gamma = 1.20$

$\zeta$	a = 1.50 b = 0.020	a = 2.00 b = 0.020	a = 2.50 b = 0.020	a = 1.50 b = 0.030	a = 2.00 b = 0.030	a = 2.50 b = 0.030	a = 1.50 b = 0.050	a = 2.50 b = 0.050	a = 3.50 b = 0.050
-0.6	+ 0.918			+ 0.861			+ 0.750		
-0.4	+ 2.774	+ 1.327		+ 2.717	+ 1.300		+ 2.663		
-0.2	+ 3.315	+ 2.810	+ 2.368	+ 3.305	+ 2.801	+ 2.359	+ 3.287	+ 2.342	+ 1.466
0	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141	+ 3.141
0.2	+ 2.627	+ 2.879	+ 3.047	+ 2.632	+ 2.884	+ 3.052	+ 2.640	+ 3.062	+ 3.263
0.4	+ 2.017	+ 2.391	+ 2.616	+ 2.022	+ 2.398	+ 2.625	+ 2.033	+ 2.642	+ 2.849
0.6	+ 1.436	+ 1.870	+ 2.118	+ 1.441	+ 1.877	+ 2.129	+ 1.449	+ 2.150	+ 2.345
0.8	+ 0.940	+ 1.399	+ 1.662	+ 0.942	+ 1.406	+ 1.673	+ 0.946	+ 1.695	+ 1.898
1.0	+ 0.542	+ 1.008	+ 1.282	+ 0.541	+ 1.013	+ 1.292	+ 0.538	+ 1.314	+ 1.538
1.2	+ 0.238	+ 0.697	+ 0.980	+ 0.234	+ 0.699	+ 0.989	+ 0.223	+ 1.007	+ 1.260
1.4	+ 0.015	+ 0.457	+ 0.746	+ 0.007	+ 0.456	+ 0.753	- 0.011	+ 0.766	+ 1.048
1.6	- 0.143	+ 0.276	+ 0.568	- 0.155	+ 0.271	+ 0.572	- 0.179	+ 0.579	+ 0.886
1.8	- 0.250	+ 0.141	+ 0.432	- 0.265	+ 0.133	+ 0.433	- 0.295	+ 0.435	+ 0.762
2.0	- 0.319	+ 0.041	+ 0.329	- 0.335	+ 0.031	+ 0.328	- 0.370	+ 0.325	+ 0.667
2.5	- 0.381	- 0.101	+ 0.168	- 0.401	- 0.118	+ 0.161	- 0.442	+ 0.145	+ 0.508
3.0	- 0.364	- 0.159	+ 0.085	- 0.382	- 0.180	+ 0.075	- 0.422	+ 0.050	+ 0.417
3.5	- 0.318	- 0.178	+ 0.042	- 0.334	- 0.200	+ 0.029	- 0.366	- 0.003	+ 0.362
4.0	- 0.267	- 0.178	+ 0.019	- 0.279	- 0.202	+ 0.005	- 0.301	- 0.032	+ 0.327
4.5	- 0.220	- 0.171	+ 0.006	- 0.228	- 0.195	- 0.009	- 0.238	- 0.050	+ 0.304
5.0	- 0.181	- 0.162	- 0.001	- 0.183	- 0.185	- 0.017	- 0.181	- 0.061	+ 0.289
6.0	- 0.122	- 0.143	- 0.007	- 0.116	- 0.165	- 0.024	- 0.088	- 0.072	+ 0.275
7.0	- 0.084	- 0.127	- 0.009	- 0.072	- 0.149	- 0.026	- 0.020	- 0.079	+ 0.275
8.0	- 0.060	- 0.116	- 0.010	- 0.043	- 0.137	- 0.027	+ 0.033	- 0.085	+ 0.287
9.0	- 0.044	- 0.108	- 0.010	- 0.023	- 0.128	- 0.027	+ 0.070	- 0.093	+ 0.311
10.0	- 0.034	- 0.101	- 0.010	- 0.009	- 0.123	- 0.027	+ 0.103	- 0.103	+ 0.351

Table 2. Ratio  $F_{1e}/F_1^*$  of the Transverse Force  $F_{1e}$  on the Nozzle Wall, per Unit Axial Length, at the Nozzle Exit Plane, Relative to the Corresponding Force  $F_1^*$  at the Throat

	$F_{1e}/F_1^*$
$a = 1.50, b = 0.020$	$- 2.468 \times 10^{-3}$
$a = 2.00, b = 0.020$	$- 3.811 \times 10^{-2}$
$a = 2.50, b = 0.020$	$- 4.730 \times 10^{-3}$
$a = 1.50, b = 0.030$	$+ 9.298 \times 10^{-3}$
$a = 2.00, b = 0.030$	$- 4.230 \times 10^{-2}$
$a = 2.50, b = 0.030$	$- 1.205 \times 10^{-2}$
$a = 1.50, b = 0.050$	$+ 3.267 \times 10^{-2}$
$a = 2.50, b = 0.050$	$- 3.279 \times 10^{-2}$
$a = 3.50, b = 0.050$	$+ 1.116 \times 10^{-1}$

Note: For  $\gamma = 1.20$ , and zero nozzle divergence angle at the exit plane.





## REFERENCES

1. Flandro, G. A., "Interaction of Inertial Waves in a Spinning Solid Propellant Rocket Motor with Spacecraft Motion," Air Force Rocket Propulsion Laboratory Report, October 1982. Also, "Fluid Mechanics of Spinning Rockets," Air Force Rocket Propulsion Report TR-86-072, January 1987.
2. Meyer, R. X., "Convective Instability in Solid Propellant Rocket Motors," AAS Paper 83-368, Advances in the Astronautical Sciences, Vol. 54, 1983.
3. Walters, A. G., "Nonsymmetric Flow in Laval Type Nozzles," Royal Soc. Phil. Transactions, A Vol. 273, 1972, pp. 185-235.
4. Darwell, H. M. and G. F. P. Trubridge, "Design of Rocket Nozzles to Reduce Gas Misalignment," J. of Spacecraft and Rockets, Vol. 5, No. 1, 1968, pp. 36-41.
5. Hoffman, J. D. and A. R. Maykut, "Gas Dynamic Gain of Supersonic Thrust Nozzles," J. of Spacecraft and Rockets, Vol. 11, No. 10, 1974, pp. 697-704.
6. Liepmann, H. W. and A. Roshko, Elements of Gas Dynamics, John Wiley and Sons, Inc., Chapters 2 and 5, 1957.



# NOMENCLATURE

$a, b$	parameters [Eq. 14)]
$F_1(z)$	transverse force on nozzle, per unit axial length
$g_1, k_1$	functions characterizing transverse pressure gradient [Eqs. (7) and (8)]
$h$	enthalpy
$L_{1y}(z)$	transverse component of angular momentum
$M_0(z)$	Mach number of zero-order (unperturbed) flow
$p$	pressure
$P_{1x}(z)$	transverse component of momentum
$R(z)$	half-width of rectangular nozzle, and radius of axisymmetric nozzle
$u, v, w$	Cartesian velocity components
$x, y, z; r, \phi, z$	Cartesian and cylindrical coordinates
$\gamma$	ratio of specific heats
$\xi, \zeta$	nondimensional lengths [Eq. (8)]
$\rho$	density
$\sigma$	$\sigma = 1$ for rectangular nozzle; $\sigma = 2$ for axisymmetric nozzle
$( )_0$	zero-order term
$( )_1$	first-order asymmetric term
$( )^*$	condition at $M_0 = 1$
$( )_i$	initial condition
$( )_e$	nozzle exit plane condition

